

Sheet (3) - Prob 1

A 2mc positive charge is isolated in a vacuum at $P_1(3, -2, -4)$ and 5mc negative charge at $P_2(1, -4, 2)$

(a) Find the vector force on negative charge

(b) " " magnitude of force on charge at P_1

Sol

$$\vec{F}_{12} = 9 \times 10^9 \times \frac{q_1 q_2}{r^2} \vec{a}_{R_{12}} \quad (1, -4, 2) \quad \vec{F}_{12} \quad + (3, -2, -4) \quad \vec{a}_1$$

$$= \frac{9 \times 10^9 \times (2 \times 10^{-3}) \times (-5 \times 10^{-3})}{r^2} \vec{a}_{R_{12}} \quad r = |\vec{R}_{12}|$$

$$\vec{R}_{12} = \vec{R}_2 - \vec{R}_1 = (1, -4, 2) - (3, -2, -4) = (-2, -2, 6) = -2\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z$$

$$|\vec{R}_{12}| = r = \sqrt{(-2)^2 + (-2)^2 + (6)^2} = \sqrt{44}$$

$$\vec{a}_{R_{12}} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$

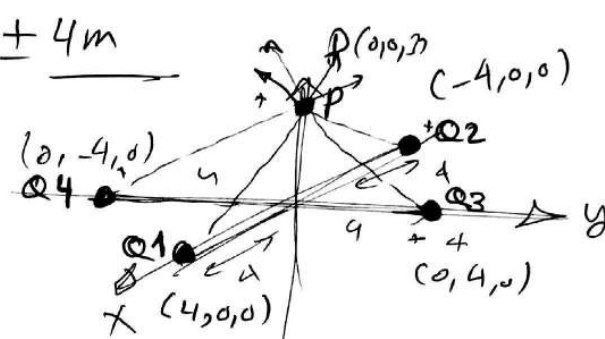
$$\vec{F}_{12} = \frac{9 \times 10^9 \times 2 \times 10^{-3} \times -5 \times 10^{-3}}{(\sqrt{44})^2} \cdot (-2\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z)$$

$$\vec{F}_{12} = 0.616 \hat{a}_x + 0.616 \hat{a}_y - 1.84 \hat{a}_z$$

(b) Magnitude = $|\vec{F}_{12}| = \sqrt{(0.616)^2 + (0.616)^2 + (-1.84)^2} = 2.04 \text{ N}$

EX(3)
Sheet(3-Prob(2))

Find force on $100 \mu\text{C}$ charge at $(0,0,3)\text{m}$
if four like charges of $20 \mu\text{C}$ are located on
x and y axis at $\pm 4\text{m}$



Sol

$$\vec{F}_t = \frac{Q_1 Q_P}{r_1^2} \times 9 \times 10^9 \hat{a}_{r_1} + \frac{Q_2 Q_P}{r_2^2} \times 9 \times 10^9 \hat{a}_{r_2} + \frac{Q_3 Q_P}{r_3^2} \times 9 \times 10^9 \hat{a}_{r_3} + \frac{Q_4 Q_P}{r_4^2} \times 9 \times 10^9 \hat{a}_{r_4}$$

$\therefore Q_1 = Q_2 = Q_3 = Q_4 = 20 \times 10^{-6}$

$$\therefore \vec{F}_t = \frac{20 \times 10^{-6} \times 100 \times 10^{-6}}{9 \times 10^9} \left[\frac{\hat{a}_{r_1}}{r_1^2} + \frac{\hat{a}_{r_2}}{r_2^2} + \frac{\hat{a}_{r_3}}{r_3^2} + \frac{\hat{a}_{r_4}}{r_4^2} \right]$$

* $\hat{a}_{r_1} = \hat{R}_P - \hat{R}_1 = (-4, 0, 3) = -4\hat{a}_x + 3\hat{a}_z$
 $|\hat{R}_1| = r_1 = \sqrt{(-4)^2 + (3)^2} = 5$ $\hat{a}_{r_1} = \frac{-4}{5}\hat{a}_x + \frac{3}{5}\hat{a}_z$ (I)

* $\hat{a}_{r_2} = \hat{R}_P - \hat{R}_2 = (4, 0, 3) = 4\hat{a}_x + 3\hat{a}_z$
 $r_2 = 5$ $\hat{a}_{r_2} = \frac{4}{5}\hat{a}_x + \frac{3}{5}\hat{a}_z$ (II)

* $\hat{a}_{r_3} = \hat{R}_P - \hat{R}_3 = (0, -4, 3) = -4\hat{a}_y + 3\hat{a}_z$
 $r_3 = 5$ $\hat{a}_{r_3} = \frac{-4}{5}\hat{a}_y + \frac{3}{5}\hat{a}_z$ (III)

* $\hat{a}_{r_4} = \hat{R}_P - \hat{R}_4 = (0, 4, 3) = 4\hat{a}_y + 3\hat{a}_z$
 $r_4 = 5$ $\hat{a}_{r_4} = \frac{4}{5}\hat{a}_y + \frac{3}{5}\hat{a}_z$ (IV)

$$\vec{F}_t = \frac{9 \times 10^9 \times 20 \times 10^{-6} \times 100 \times 10^{-6}}{(5)^2} [4 \times \frac{3}{5} \hat{a}_z] = 1.73 \hat{a}_z \text{ (N)}$$

EX(4)

Sheet (3) Prob (3)

Two point charges, $Q_1 = 50 \mu\text{C}$, $Q_2 = -10 \mu\text{C}$
located at $P_1(-1, 1, -3)$ & $P_2(3, 1, 0)$ Find force on Q_1

Report

EX(5) sheet (3) Prob (4)

Quiz مراجعة

Point charge $Q_1 = 300 \mu\text{C}$, located at $(1, -1, -3) \text{ m}$

experiences a force $\vec{F}_1 = 8\hat{a}_x - 8\hat{a}_y + 4\hat{a}_z \text{ N}$
(exerts) \vec{F}_1 on Q_1

due to Q_2 located at $(3, -3, -2)$, Find Q_2

Sol $\vec{F}_{21} = \frac{Q_1 Q_2}{r^2} \cdot 9 \times 10^9 \times \hat{a}_R = 8\hat{a}_x - 8\hat{a}_y + 4\hat{a}_z$

$$\Rightarrow \vec{R}_{21} = \vec{R}_1 - \vec{R}_2 = (-2, 2, -1)$$
$$= -2\hat{a}_x + 2\hat{a}_y - \hat{a}_z$$

$$|\vec{R}_{21}| = \sqrt{(-2)^2 + (2)^2 + (-1)^2} = 3$$

$$\hat{a}_R = -\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y - \frac{1}{3}\hat{a}_z$$



$$\therefore 8\hat{a}_x - 8\hat{a}_y + 4\hat{a}_z = \frac{9 \times 10^9 \times 300 \times 10^{-6} \times Q}{(3)^2} \cdot \left[-\frac{2}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{1}{3}\hat{a}_z \right]$$

الآن نحلل طرف اليمين ونحلل طرف اليسار ^{من أجل} \hat{a}_x ونكافئ

$$\therefore 8\hat{a}_x = \frac{9 \times 10^9 \times 300 \times 10^{-6} \times Q}{9} \cdot \left(-\frac{2}{3} \right) \hat{a}_x$$

$$\therefore Q = -$$

حل مسألة مرسومة في ورقة

Sheet(3) Prob(5) → Point charge

Find electric field intensity at Point (2,1,3) due to 2 charges of $Q_1 = 5 \mu\text{C}$ and $Q_2 = 8 \mu\text{C}$ at points (3,1,2) and origin

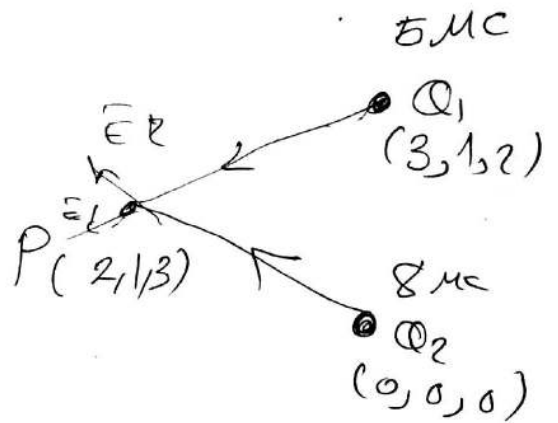
س

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Q_1}{4\pi\epsilon R_1^2} \hat{a}_{R_1} + \frac{Q_2}{4\pi\epsilon R_2^2} \hat{a}_{R_2}$$

$$= \frac{Q_1}{4\pi\epsilon R_1^3} \vec{R}_1 + \frac{Q_2}{4\pi\epsilon R_2^3} \vec{R}_2$$

$$= \frac{5 \times 10^{-6}}{R_1^3} \times 9 \times 10^9 \vec{R}_1 + \frac{9 \times 10^9 \times 8 \times 10^{-6}}{R_2^3} \vec{R}_2$$



$$* \vec{R}_1 = (2-3, 1-1, 3-2) = -a\hat{x} + a\hat{z}$$

$$|\vec{R}_1| = r_1 = \sqrt{2}$$

$$* \vec{R}_2 = (2-0, 1-0, 3-0) = 2a\hat{x} + a\hat{y} + 3a\hat{z}$$

$$|\vec{R}_2| = r_2 = \sqrt{14}$$

$$\vec{E}_p = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{(\sqrt{2})^3} [-a\hat{x} + a\hat{z}] + \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(\sqrt{14})^3} [2a\hat{x} + a\hat{y} + 3a\hat{z}]$$

$$= \text{○} a\hat{x} + \text{○} a\hat{y} + \text{○} a\hat{z}$$

(give)

Sheet (3) → No (6)

An electron Beam may be Approximated by a right circular cylinder of Random Ⓜ that contains a volume charge density $\rho_v = \frac{k}{c+r^2} \text{ C/m}^3$
Evaluate the total charge per unit length of beam



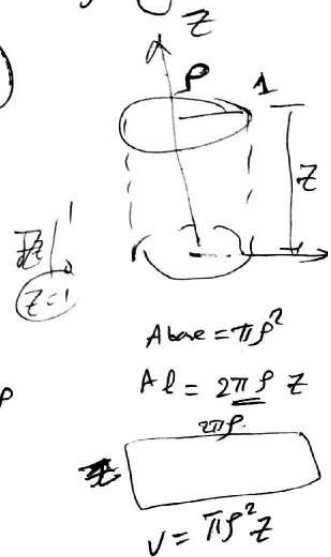
$$Q = \int \rho_v dv$$

For cylindrical $dv = \rho d\rho d\phi dz$

$$\therefore Q = \int_0^z \int_0^{2\pi} \int_0^f \frac{k}{c+r^2} \cdot \rho d\rho d\phi dz$$

$$= (2\pi) k \int_0^z \int_0^f \frac{\rho}{c+\rho^2} d\rho dz = 2\pi k \int_0^z \left[\ln(c+\rho^2) \right]_0^f dz$$

$$= 2\pi k z \left[\ln(c+f^2) - \ln c \right]$$



$$Q = \pi k \ln \frac{c+r^2}{c}$$

$$Q = \pi k \ln \frac{c+r^2}{c}$$

Sheet (3) No (7)

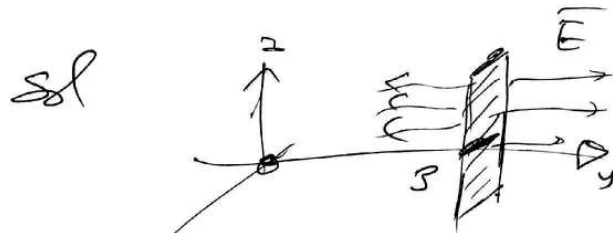
[7] A plane $y=3m$, contains a uniform charge distribution of density $\rho_s = \left(\frac{10^{-8}}{6\pi}\right) C/m^2$, Determine

E at all points

Sol

$E (y > 3)$

$$= \frac{\rho_s}{2\epsilon_0} \cdot \hat{a}_n \rightarrow \hat{a}_y$$



$$= \frac{10^{-8}/6\pi}{2 \times 8.85 \times 10^{-12}} \hat{a}_y = 30 \hat{a}_y$$

$\rightarrow \sqrt{36\pi}$

$E (y < 3)$

$$= \frac{\rho_s}{2\epsilon_0} \cdot \hat{a}_n \rightarrow -\hat{a}_y = -30 \hat{a}_y$$

Sheet (3) - No (8)

determine \vec{E} at $(x, -1, 0)$ due to uniform sheet charge with $\rho_s = \frac{1}{3\pi} \text{ nC/m}^2$, is located at $z = 5 \text{ m}$

& Uniform line charge with $\rho_L = -\frac{25}{9} \text{ nC/m}$

at $z = +3, y = 3$, (نقطه الشحنة الخطية)

حل

$$\vec{E} = \vec{E}_{\text{line}} + \vec{E}_{\text{sheet}}$$

$$\vec{E}_s = \frac{\rho_s}{2\epsilon} \hat{a}_n$$

(نصف المساحة من كل اتجاه)
 $\hat{a}_z \text{ or } -\hat{a}_z$ في $z = 5$

$$\therefore \vec{E}_s = \frac{(\frac{1}{3\pi} \times 10^{-9})}{2 \times 8.85 \times 10^{-12}} \cdot [-\hat{a}_z] \rightarrow \textcircled{1}$$

$$\vec{E}_l = \frac{\rho_L}{2\pi\epsilon r} \hat{a}_r \rightarrow$$

$$\vec{R} = (x, -1, 0) - (x, 3, 3) = (0, -4, -3)$$

$$= -4\hat{a}_y - 3\hat{a}_z$$

$$|\vec{R}| = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\therefore \vec{E}_l = \frac{(-\frac{25}{9} \times 10^{-9})}{2\pi \times 8.85 \times 10^{-12}} \cdot \frac{(-4\hat{a}_y - 3\hat{a}_z)}{5} \rightarrow \textcircled{2}$$

$$\vec{E} = -8\hat{a}_y - 12\hat{a}_z$$

الجواب

